Development of Brain-Computer Interface Systems

Jeremy Hill, Jason Farquhar[1], Moritz Grosse-Wentrup, Suzanne Martens, Bernhard Schölkopf

Brain-computer interface (BCI) research aims to construct systems that could allow a paralysed person to communicate, by decoding the user's intentions directly from brain signals. Currently, there is still a gap between BCI's proven success with healthy subjects, (and even subjects with restricted motor control) and the lack of cases in which BCI has been shown to be effective in a clinical setting, as a "neural prosthesis" for completely paralysed users who have no other means of communication.

One reason may be the unsuitability of popular *induction* strategies (i.e. the types of stimuli and/or mental tasks used to elicit measurable differences in brain signal patterns) for completely paralysed users. Here we have been active in developing experimental approaches that are based only on auditory stimuli [1], or only tactile stimuli [2], exploiting modulation of evoked responses by shifts in the user's focus of attention.

Another area of advancement is that of *measurement* technology – for example the use of implanted electrocorticographic (ECoG) electrodes. Here we have been cooperating intensively with the department of Neurosurgery and the department of Medical Psychology in the University of Tübingen to attempt to develop a communication system for a completely paralysed user (work in progress – see Figure 1).

The third area of focus is the investigation and improvement of machine-learning algorithms for the *decoding* of brain signals. We have been comparing the suitability of different standard algorithmic approaches for different measurement technologies [3, 4] as well as developing tailor-made algorithms for specific BCI contexts. One particular approach we have developed is to unify the processes of *feature extraction and classification* by treating feature extraction tuning as hyper-parameter learning. This allows us to optimize the classifier's measure of generalization performance directly and improve overall BCI system performance. We demonstrated the idea with 2 well-known classifiers, the SVM and the Gaussian process, applied to detecting event-related desynchronization in imagined-movement experiments with band-power features. In this way we have simultaneously learned the temporal window, frequency band and spatial filter [5]. In future we will develop feature representations based on causality and network information theory, to provide additional information for decoding (see [p. 51]).

Finally, we are interested in the ways in which information theory can help us in *encoding* communication intentions most efficiently. Here we have been able to show that the use of error-correcting codes can bring a benefit in the realization of a BCI speller such as that depicted in Figure 2, but that this interacts with the properties of the stimulus being used [6, 7] – see [p. 50]

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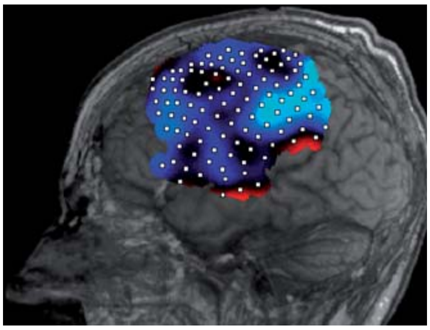
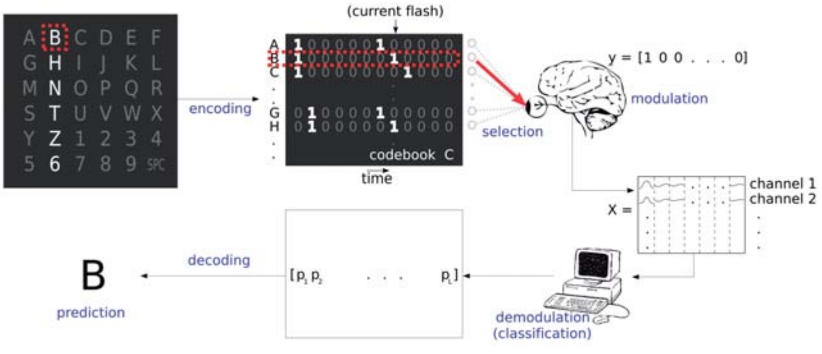


1. Locus of the modulation of 12.5Hz ECoG signals in a completely paralysed patient during an imagined spatial navigation task.



2. Schematic illustration of a visual BCI speller system: traditionally, one row or one column of letters flashes at any one time – however, this leads to a poor *codebook* from an information-theoretic perspective. Improvements are possible, but only if one also respects certain constraints on the stimulus properties and timing (see [p. 50]).

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Brain-Computer Interface Speller Systems for Locked-In Patients

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Patients in a locked-in state (LIS) are subject to complete paralysis of nearly all voluntary muscles in their body and, as a consequence, lose the ability to communicate. A brain-computer interface (BCI) speller system is potentially useful for LIS patients since it may provide them with a new communication channel. The visual speller system based on the P300 event-related potential [1] is one of the BCI speller systems currently used. The user has the choice to select letters from a letter grid presented on a pc screen in the following way (see also Figure 2 in [p. 49]). A stream of stimulus events takes place on the columns and the rows of the letter grid such that each letter is unambiguously encoded. A codebook can be thought of as a collection of codewords, each of which is a binary string representing the stimulus events over time for one letter. By focusing his attention on one letter and producing attention modulated brain signals in response to the stimulus events on his letter (target events), the user can transmit the corresponding codeword information. A classifier is trained to discriminate between the attention modulated brain signals and the "common" brain signals. The resulting classifier outputs can be matched to one of the codewords in order to predict the letter. One of the challenges is to improve the bitrate of the system such that users can spell more words per minute without a loss in letter prediction accuracy.

A second project involves the improvement of a recently proposed auditory version of the visual P300 speller [2]. This system would allow locked-in patients with deficiencies in the visual system to use the speller system. In the current implementation, the user has to relate 2 numbers to each letter indicating the row and column index of the letter grid. However, many subjects do not reach satisfactory letter prediction accuracies. Insight into the neurophysiology of the brain signals may open pathways for improving the speller system. We found evidence for overlap and refractory effects in the visual P300 speller data [3]. These effects occur whenever two stimulus events on the letter of interest occur within a small time span. These neurophysiologic effects affect the classification performance significantly (see Figure 1). By means of a simulation model, we were able to quantify the contributions of the overlap and refractory effects. We also found that a different stimulus type suffered less from refractory effects. This information can be used in the codebook design, see [4]. A next step is to design algorithms that deal with the overlap effects in visual speller data. We hope to improve the auditory speller performance by reducing the task complexity for the user. We plan to design a "user-friendly" encoding in which there is direct relation between the stimulus events and the letters. This is work in progress.

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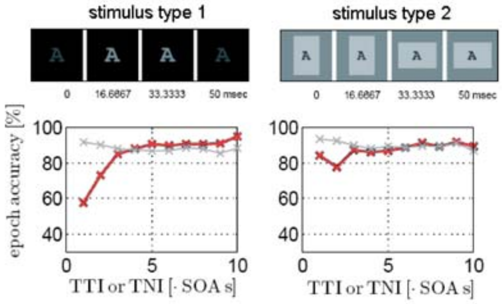
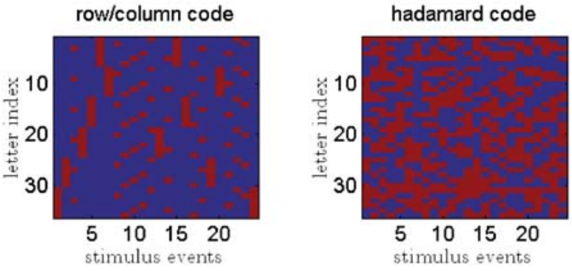


1. Illustration of the two stimulus types used in our experiments. A stimulus event consists of an illumination of one or more letters (stimulus type 1) or a flipping of the grey box surrounding the letters (stimulus type 2). The bottom plots show the classification accuracy for both stimulus types as a function of the target-to-target interval (TTI) or analogously the target-to-non-target interval (TNI), which is the temporal interval between the current and a former stimulus event (red curves), expressed as a number of Stimulus Onset Asynchronies (SOAs) or base periods of the stimulus cycle. The grey curves may be ignored here. Notice that there is an interaction with the physical characteristics of the stimulus: stimulus type 1 (left plot) shows a larger decrease in performance for small TTI than stimulus type 2 (right plot).



2. The traditional row/column stimulus pattern in P300 spellers leads to the codebook in the left panel (red = target event, blue = non-target events) with minimum Hamming distance 4. Better codes are possible (Hadamard code, right, minimum Hamming distance 12) but note the increased target frequency: the results in Figure 1 suggest this would weaken the discriminability of the individual brain signals. How can we find the optimal tradeoff?

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Network Connectivity Analysis for Non-invasive Brain-Computer Interfaces

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Brain-Computer Interfaces (BCIs) aim to provide a means of communication for subjects suffering from diseases of the peripheral nervous system such as amyotrophic lateral sclerosis (ALS). While satisfactory peformance of non-invasive BCIs has been demonstrated with healthy subjects and subjects in early to middle stages of ALS, so far no communication could be established with a completely locked-in subject unable to communicate by other means.

One potential reason for this failure is an insufficient capability of subjects in late stages of ALS to intentionally modulate local bandpower changes of the electromagnetic field of the brain, which constitutes the type of feature space commonly employed in non-invasive BCIs. The goal of this project is to develop a new feature extraction method for non-invasive BCIs based on connectivity measures between brain regions. In principle, this type of feature space is independent of local bandpower changes, and thus might enable communication with subjects in late stages of ALS.

The primary challenge in this project is to define a suitable metric for brain connectivity as measured by EEG/MEG. We currently develop an approach for connectivity analysis based on concepts of network information theory. Given an observed distribution of information between time series recorded from different brain regions, we aim to determine the graph with the lowest information transmission capacity capable of explaining the observed distribution of information (Figure 1). In comparison to commonly used metrics for brain connectivity (cf. [1]), the primary advantage of this approach is that it does not require any assumptions on the underlying model generating the observed time series.

We could already show in [2] that in healthy subjects, motor imagery of either hand induces changes in connectivity between primary motor, premotor, sensorimotor, and frontal cortex that occupy different spectral bands than those associated with local bandpower changes (Figure 2). This indicates that connectivity based features provide information not contained in local bandpower changes, thereby supporting our hypothesis that connectivity measures constitute a new type of feature space that might be suitable for subjects in late stages of ALS.

However, the experimental evaluation in [2] was carried out using the concept of Transfer Entropy which only applies to bivariate time series. Further, only experimental data obtained from healthy subjects was utilized. In future work, we will extend our methodology to connectivity measures based on network information theoretic concepts, and test this approach on experimental data obtained from completely locked-in subjects in late stages of ALS.

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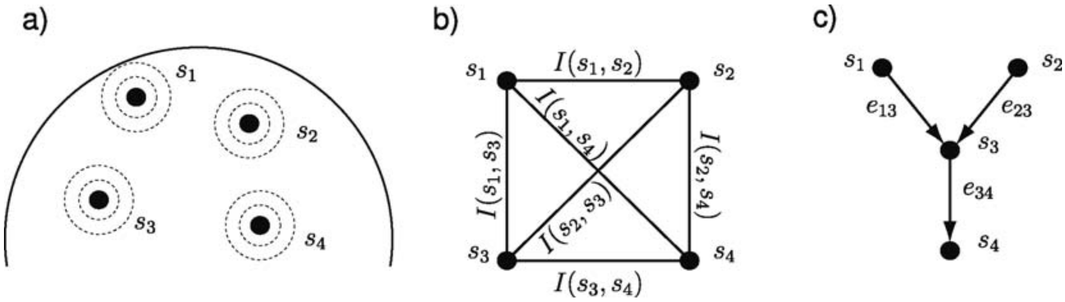
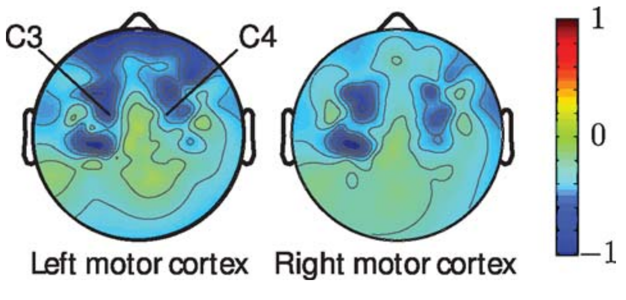


1. Illustration of the proposed concept of network connectivity analysis. a) Estimated EEG sources. b) Observed distribution of information between sources. c) Inferred network with minimum edge capacities e_{ij}.



2. Change of normalized Transfer Entropy between rest and motor imagery of either hand from all recording sites into the left/right motor cortex in the γ-band [2]. C3/C4 marks position of left/right primary motor cortex.

The Optimal Stimuli for Free-viewing Natural Images are Center-surround Filters

Wolf Kienzle[1], Matthias O. Franz[2], Bernhard Schölkopf, Felix A. Wichmann[3]

In which way do the local image statistics at the center of gaze differ from those at randomly chosen image locations? In 1999, Reinagel and Zador [1] showed that RMS contrast is significantly increased around fixated locations in natural images. Since then, numerous additional hypotheses have been proposed, based on edge content, entropy, self-information, higher order statistics, or sophisticated models such as that of Itti and Koch [2]. Unfortunately, while these models are rather different in nature, they hardly differ in terms of their preditive power. This complicates the question of what drives human eye movements. To shed light on this problem, we analyze the nonlinear receptive fields of an eye movement model which is purely data-driven [3]. It consists of a nonparametric radial basis function network, fitted to human eye movement data. To avoid a bias towards specific image features such as edges or corners, we deliberately chose raw pixel values as the input to our model, not the outputs of some filter bank. The learned model is analyzed by computing its optimal stimuli. It turns out that there are two maximally excitatory stimuli, both of which have center-surround structure (see Figure). We argue that these can be seen as nonlinear receptive fields of the underlying system. In particular, we show that a small radial basis function network with the optimal stimuli as centers predicts unseen eye movements as precisely as the full model.

The fact that center-surround filters emerge from a simple optimality criterion – without any prior assumption that would make them more probable than e.g. edges, corners, or any configuration of pixels values in a square patch – suggests a special role of these filters in free-viewing of natural images. Also, since our simple network performs as good as Itti and Koch's method, which is based on an elaborate model of the human visual system, we conjecture that the basic result by Reinagel and Zador has not been improved substantially so far. Rather, the similar performance of seemingly different models might be explained by the fact that every one of them is correlated with the same type of contrast, implemented by simple center-surround filters.

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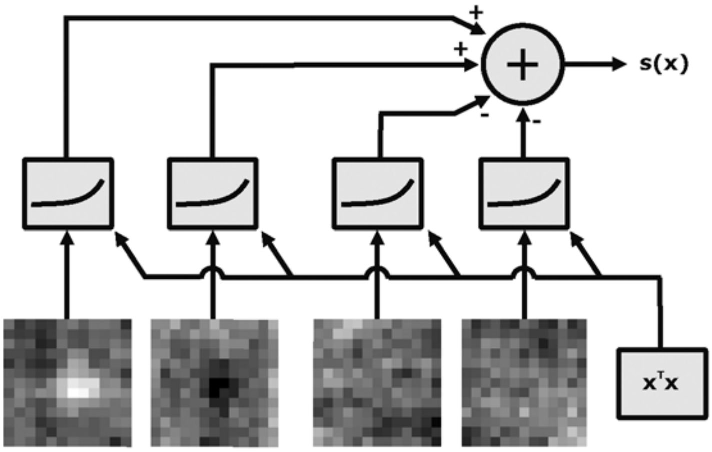

The proposed saliency model. Its front-end consists of the optimal stimuli from our analysis. The two filter masks on the left are the maximally excitatory stimuli found in our experiment, the two filters on the right denote the most inhibitory stimuli.

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Classification of Echolocation Signals

Matthias O. Franz[1], Yossi Yovel[2], Peter Stilz[2], Hans-Ulrich Schnitzler[2]

Classification of plants according to their echoes is an elementary component of bat behavior that plays an important role in spatial orientation and food acquisition. Vegetation echoes are, however, highly complex stochastic signals: from an acoustical point of view, a plant can be thought of as a three-dimensional array of leaves reflecting the emitted bat call. The received echo is therefore a superposition of many reflections.

Our experiments suggest that the classification of these echoes might not be such a troublesome task for bats as formerly thought. We use a rather simple approach to classifying signals from a large database of plant echoes that were created by ensonifying plants with a frequency-modulated bat-like ultrasonic pulse. Our algorithm uses the spectrogram of a single echo from which it only uses features that are undoubtedly accessible to bats. A standard Support Vector Machine classifier automatically extracts suitable linear combinations of time and frequency cues from the spectrograms which allow for classification with a surprisingly high accuracy. This demonstrates that ultrasonic echoes are highly informative about the species membership of an ensonified plant, and that this information can be extracted with a rather simple, biologically plausible analysis method. Thus, our findings provide a new explanatory basis for the poorly understood abilities of bats in classifying vegetation and other complex objects [1].

In order to find a theoretical basis for these results we computed the power spectrum of a large database of ultrasonic plant echo envelopes. On a bi-logarithmic plot (Figure), the averaged power spectra of all plants have a similar sigmoid shape with three approximately linear domains that represent different scales of structure and differ between species. We hypothesize that the first domain is influenced by the gross skeleton of branches, while the others are associated with smaller scale structures. In control experiments we compared the power spectrum of a single leaf, a branch and a group of branches, and tested the effect of systematically decreasing the leaf density of a plant. We found a similar dependence between leaf density and power spectrum as predicted from our hypothesis. In addition, modeling plants as simple 3D textures with stationary statistics was sufficient to predict the characteristic shape of the spectra. Altogether, these findings indicate that the biologically plausible spectrotemporal decomposition used in our approach is directly related to relevant morphological plant descriptors such as leaf and branch density or size, and thus explains to a large extent the surprising simplicity of ultrasonic plant classification.

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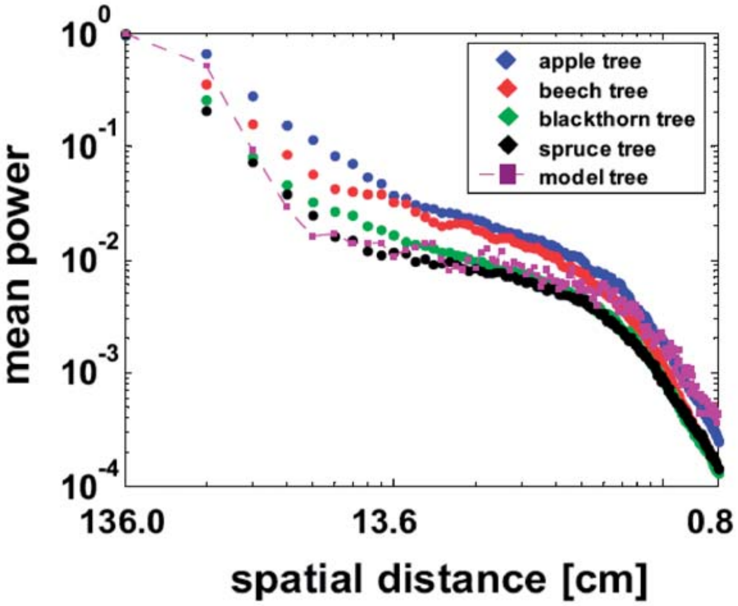

Averaged power spectra corresponding to each of the different kinds of plants.

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Towards Quantitative PET/MR: Machine Learning Methods for MR-based Attenuation Correction

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For quantitative PET information, correction of tissue photon attenuation is mandatory. Usually in conventional PET, the attenuation map is obtained from a transmission scan, which uses a rotating radionuclide source, or from the CT scan in the case of a combined PET/CT scanner. In the case of PET/MR scanners, currently under development, there is insufficient space for the rotating source and one wants to calculate the attenuation map from the MR image instead. This task is challenging since MR intensities correlate with proton densities and tissue relaxation properties, rather than with attenuation-related mass density [1]. We have developed a novel method that combines recognition of local patterns and atlas-registration, which captures global variation of anatomy [2, 3, 4]. From a given MR image, our method predicts pseudo-CT images which are then used for attenuation correction as in a PET/CT scanner.

For human brain scans, we have shown on a database of 17 MR-CT image pairs that our method reliably enables estimation of a pseudo-CT image from the MR image alone. On additional datasets of MR/CT/PET triplets of human brain scans we have compared MR based attenuation correction with CT based correction. Our approach enables PET quantification with a mean error of 3.2% for predefined regions of interest, an error found to be clinically insignificant. However, our method is not specific to brain imaging and we have achieved promising initial results on whole body data sets [5].

Our method requires accurately registered MR-CT images for training. For this purpose we are working on learning a better similarity measure between images: we learn a function $f(x, y)$ that characterizes the similarity between the input x and the output y where x and y are image patches extracted from the two different modal images. In particular, we use max-margin structured prediction methods that incorporate a kernel defined jointly over the input and output space. This method allows us to incorporate the cost function into the training, where the cost is defined as the spatial distance between output patches. We define the final similarity measure as the sum of the similarity scores of patch pairs in the images.

Ongoing work focuses on validating the method for whole-body imaging. We are closely collaborating with the University Hospital Tübingen and with Siemens Medical and our method is now being tested on clinical images obtained with the world-wide first prototype clinical PET/MR scanner.

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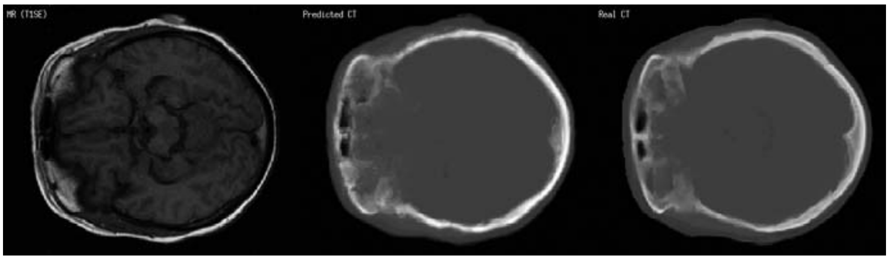
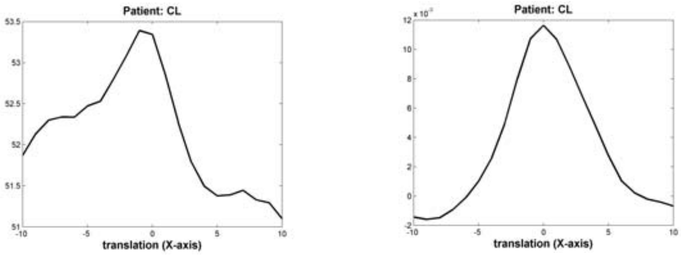


Left: The patient MR image. Middle: The “pseudo-CT” as predicted using our method. Right: The real CT image.



Registration functions for MR and CT matching when translating along an in-plane axis from zero (optimal matching). Left: normalized mutual information, Right: the proposed max-margin measure.

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Optimization of Magnetic Resonance Imaging Sequences by Bayesian Experimental Design

Matthias Seeger[1], Hannes Nickisch, Rolf Pohmann, Bernhard Schölkopf

MR images are acquired by sampling along smooth trajectories in *k*-space (two-dimensional Fourier space) (see Figure 1 and 2). Acquisition could be made more efficient, and fMRI temporal resolution improved, by undersampling this space with as little redundancy as possible. Nonlinear sparse methods can reconstruct images at a desired spatial resolution even from an undersampling [1], by exploiting recent knowledge about natural image statistics and convex optimization. Ideally, measurement designs should be optimized (partially) automatically, so that researchers with little experience in MRI sequence design can adapt these control flows to their needs. A promising approach is Bayesian sequential experimental design, recently extended to sparse models [2], provided that computation time on full MR images can be scaled to an acceptable level.

We proposed a novel algorithm for Bayesian sequential optimization of measurement architectures for natural images, which runs orders of magnitude faster than previous methods. [p. 31] Our plan is to apply this technology to optimize trajectories of MRI sequences. Results of a preliminary study for spiral MRI trajectories (Figure 3) are promising [3].

For the study in [3], we acquired human brain slices with a TSE sequence (Siemens 3T scanner), interpolating spiral measurements from this data. The images reconstructed from the full data served as ground truth. Our Bayesian method was used to optimize offset angles of Archimedian spiral interleaves, and sparse maximum a posteriori (MAP) reconstruction [1] results based on the optimized trajectories were compared against regular symmetric spacing and randomly drawn angles, the setups most commonly used today.

The performance of MAP reconstruction depends significantly on the measurement design (set of offset angles) used: equispaced angles (eq; symmetric from 0 to 2π) are outperformed by uniform random angles (rd), but these random designs can be strongly improved upon by measurements optimized with our Bayesian method (op). Reconstruction errors are shown in Figure 3.

In future work, we plan to address measurement optimization for Cartesian 3D trajectories of fMRI sequences and high-field NMR spectroscopy.

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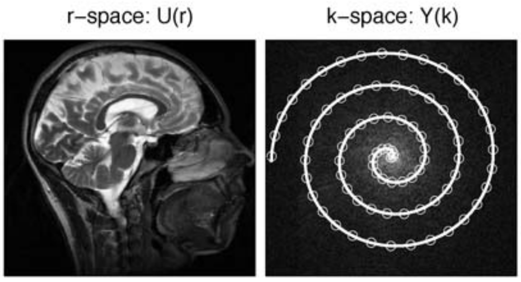

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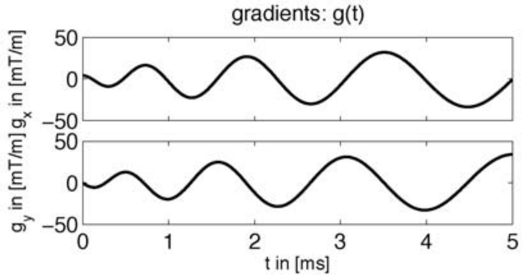
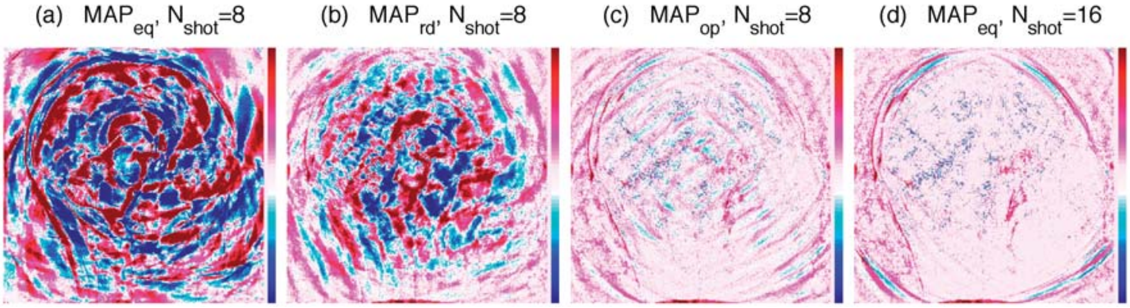


2. Gradients.



3. Differences of MAP reconstruction to true image (upper left) (a) using 8 interleaves with equispaced offset angles in (0, 2π); (b) using 8 interleaves with randomly drawn angles; (c) using 8 interleaves with optimized angles; (d) using 16 interleaves with regular Nyquist spacing.

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